可见光通信网络信道容量和干扰管理

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Contents

Introduction

- Use the visible light as the information carrier, LED as the transmitter and Photon Detector(PD) as the receiver;
- Vast license-free frequency spectrum;
- No electromagnetic interference;
- Inherent security;
- High data rate, 96 Mbps in IEEE 802.15.7, Gbps boost data rates in some research.

Introduction

VLC signal features

- Intensity modulation and direct detection ⇒ **real and non-negative** ;
- Eye safety ⇒ **peak optical power (amplitude) limited** ;
- Practical illumination ⇒ **average optical power limited**.

- Shannon Formula and Gaussian distribution cannot be used for VLC.

Motivation

Opening Problem of VLC: capacity and input distribution

- The capacity-achieving distribution of VLC is **discrete over a finite set of points**[1]
- The number of the discrete points, the amplitude of the discrete points and the probabilities of the discrete points are unknown;
- **Exhaustive Search**, without analytical expression.[1,2]
- Some recent literatures have spent efforts on investigation how to approximate the channel capacity of VLC, i.e., upper bounds or lower bounds;

[1] J. G. Smith, "The information capacity of amplitude- and variance- constrained scalar gaussian channels," Inf. Contr., vol. 18, no. 3, pp. 203-219, Feb. 1971.

[2] T. Chan, S. Hranilovic, and F. Kschischang, "Capacity-achieving probability measure for conditionally gaussian channels with bounded inputs," IEEE Trans. Inf. Theory, vol. 51, no. 6, pp. 2073–2088, Jun 2005.

Motivation

Upper bound of VLC Capacity

- Duality-based approach[3]
- Sphere packing method[4]
- Sphere packing method[5]
- Sphere packing based Recursive approach[6]

[3] A. Lapidoth, S. M. Moser, and M. Wigger, "On the capacity of free-space optical intensity channels," IEEE Trans. Inf. Theory, vol. 55, no. 10, pp. 4449–4461, Oct. 2009.

[4] A. A. Farid and S. Hranilovic, "Capacity bounds for wireless optical intensity channels with gaussian noise," IEEE Trans. Inf. Theory, vol. 56, no. 12, pp. 6066–6077, Dec. 2010.

[5] J. B. Wang, Q. S. Hu, J. Wang, M. Chen, and J. Y. Wang, "Tight bounds on channel capacity for dimmable visible light communications," J. Lightw. Technol., vol. 31, no. 23, pp. 3771–3779, Dec. 2013.

[6] A. Chaaban, J. Morvan, and M. Alouini, "Free-space optical communications: capacity bounds, approximations, and a new sphere-packing perspective," IEEE Trans. Commun., vol. 64, no. 3, pp. 1176–1191, Mar. 2016.

Motivation

Lower bound of VLC Capacity

- Differential entropy maximization (Peak and average optical power)[7]
- Discrete entropy maximization (Average optical power)[8]
- Discrete entropy maximization (Peak and average optical power)[9]

[7] A. Lapidoth, S. M. Moser, and M. Wigger, "On the capacity of free-space optical intensity channels," IEEE Trans. Inf. Theory, vol. 55, no. 10, pp. 4449–4461, Oct. 2009.
[8] A. A. Farid and S. Hranilovic, "Capacity bounds for wireless optical intensity channels with gaussian noise," IEEE Trans. Inf. Theory, vol. 56, no. 12, pp. 6066–6077, Dec. 2010.
[9] A. A. Farid and S. Hranilovic, "Channel capacity and non-uniform signaling for free-space optical intensity channels," IEEE J. Sel. Areas Commun., vol. 17, no. 9, pp. 1553–1563, Dec. 2009.

Exact Capacity of VLC Channel

- System Model
- Inexact Gradient Projection Method
- Simulation Results
- Conclusion

Exact Capacity of VLC Channel

IM/DD Channel Model

$$Y = X + Z, \tag{2.1}$$
$$0 \leq X \leq A, \tag{2.2}$$
$$\mathbb{E}\{X\} \leq \mu, \tag{2.3}$$
$$Z \sim \mathcal{C}\left(0, \sigma^2\right). \tag{2.4}$$

$P(X)$ – the distribution of X

$$\Pr\{X = x_k\} = p_k \geq 0,\ \forall k \in \mathcal{K}, \tag{2.5a}$$
$$\mathbb{E}\{X\} = \sum_{k=1}^{K} p_k x_k \leq \mu, \tag{2.5b}$$
$$\sum_{k=1}^{K} p_k = 1, \tag{2.5c}$$
$$0 \leq x_k \leq A,\ \forall k \in \mathcal{K}, \tag{2.5d}$$

Exact Capacity of VLC Channel

Capacity

$$
\begin{aligned}
C^{\text{VLC}} &= \max_{P(X)} I\left(X;Y\right) && \text{(2.6a)}\\
&= \max_{P(X)} h\left(Y\right) - h\left(Y|X\right) && \text{(2.6b)}\\
&= \max_{P(X)} -\int_{-\infty}^{\infty} f_Y\left(y\right)\log_2 f_Y\left(y\right)\mathrm{d}y - \frac{1}{2}\log_2 2\pi e\sigma^2, && \text{(2.6c)}
\end{aligned}
$$

$f_Y\left(y\right)$ –the probability density function (PDF) of Y

$$
f_Y\left(y\right) = \frac{1}{\sqrt{2\pi}\sigma}\sum_{k=1}^{K} p_k e^{-\frac{\left(y-x_k\right)^2}{2\sigma^2}} \tag{2.7}
$$

Problem Formulation

Finding the exact capacity of VLC channel can be formulated as the following optimization problem:

$$\min_{K,\{p_k\},\{x_k\}} \int_{-\infty}^{\infty} \frac{\sum_{k=1}^{K} p_k e^{-\frac{(y-x_k)^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma} \log_2 \frac{\sum_{j=1}^{K} p_j e^{-\frac{(y-x_j)^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma} \mathrm{d}y \tag{2.8}$$

$$\text{s.t.} \Pr\{X = x_k\} = p_k \geq 0, \ \forall k \in \mathcal{K}, \tag{2.9}$$

$$\mathbb{E}\{X\} = \sum_{k=1}^{K} p_k x_k \leq \mu, \tag{2.10}$$

$$\sum_{k=1}^{K} p_k = 1, \tag{2.11}$$

$$0 \leq x_k \leq A, \ \forall k \in \mathcal{K}, \tag{2.12}$$

Concise form

Variables

$$\mathbf{x} \triangleq [x_1, \ldots, x_k, \ldots, x_K]^T,$$
$$\mathbf{p} \triangleq [p_1, \ldots, p_k, \ldots, p_K]^T,$$
$$\phi(\mathbf{p}) \triangleq \int_{-\infty}^{\infty} \mathbf{p}^T \mathbf{q}(y) \log_2 \mathbf{p}^T \mathbf{q}(y) \mathrm{d}y,$$
$$\mathbf{q}(y) \triangleq \frac{1}{\sqrt{2\pi}\sigma} \left[e^{-\frac{(y-x_1)^2}{2\sigma^2}}, \ldots, e^{-\frac{(y-x_k)^2}{2\sigma^2}}, \ldots, e^{-\frac{(y-x_K)^2}{2\sigma^2}} \right]^T.$$

Exact capacity of VLC

$$\min_{K,\mathbf{p},\mathbf{x}} \phi(\mathbf{p}) \tag{2.13a}$$
$$\text{s.t. } \mathbf{p}^T \mathbf{1} = 1, \tag{2.13b}$$
$$\mathbf{p}^T \mathbf{x} \leq \mu, \tag{2.13c}$$
$$\mathbf{p} \geq \mathbf{0}, \tag{2.13d}$$

Optimization of K and $\mathbf{x}$

Fixing $\mathbf{x}$ with Equal Spacing

Pick the K values $\{x_l\}_{1\le l\le K}$ from the range $[0, A]$ with equal spacing, i.e.,

$$x_l \triangleq \frac{A}{K-1}\left(l-1\right),\ l=1,\ldots,K. \tag{2.14}$$

Proposition 1: *Suppose that* K^* *and* $\{x_k^*\}_{1\le k\le K^*}$ *are the optimal solutions of problem* (2.13)*, where* $x_1^* \le \ldots \le x_k^* \le x_{k+1}^* \le \ldots \le x_K^*$*. Then, for a given* $\varepsilon_0 > 0$*, when* $K \ge \left\lceil \frac{A}{\varepsilon_0} \right\rceil + 1$*, there exists an equal spacing sequence* $\{x_l\}_{1\le l\le K}$ *satisfying*

$$\min_{1\le l\le K} |x_k^* - x_l| \le \varepsilon_0,\ \forall k\in\mathcal{K}^*, \tag{2.15}$$

where $\mathcal{K}^* \triangleq \{1,2,\ldots,K^*\}$.

Optimization of $\mathbf{p}$

Convex Problem with Simplex Constraints

$$\min_{\mathbf{p}} \phi(\mathbf{p}) \tag{2.16a}$$

$$\text{s.t. } \mathbf{p} \in \Upsilon, \tag{2.16b}$$

where $\Upsilon \triangleq \{\mathbf{p} | \mathbf{p}^T \mathbf{1}_K = 1, \mathbf{p}^T \mathbf{x} \leq \mu, \mathbf{p} \geq \mathbf{0}, \}$.

Gradient Projection Method = Gradient Descent + feasible Projection

Optimization of $\mathbf{p}$

Gradient Projection Method

For nth Iteration,

$$\widehat{\mathbf{p}}_{n+1} \triangleq \mathbf{p}_n - \alpha_n \nabla \widetilde{\phi} \left(\mathbf{p}_n \right), \tag{2.17a}$$

$$\mathbf{p}_{n+1} = \mathrm{Proj}_{\Upsilon} \left(\widehat{\mathbf{p}}_{n+1} \right), \tag{2.17b}$$

where $\alpha_n \in (0, 1]$ is a stepsize of the nth iteration,

$$\nabla \phi \left(\mathbf{p} \right) \triangleq \int_{-\infty}^{\infty} \left(\mathbf{q} \left(y \right) \log_2 \mathbf{p}^T \mathbf{q} \left(y \right) + \frac{1}{\ln 2} \mathbf{q} \left(y \right) \right) \mathrm{d}y, \tag{2.18}$$

$$\mathrm{Proj}_{\Upsilon} \left(\mathbf{x} \right) = \begin{cases} \mathbf{x}, & \text{if } \mathbf{x} \in \Upsilon; \\ \arg\min\limits_{\widehat{\mathbf{x}} \in \Upsilon} \left\| \mathbf{x} - \widehat{\mathbf{x}} \right\|^2, & \text{otherwise.} \end{cases} \tag{2.19}$$

Neither the objective function $\phi \left(\mathbf{p} \right)$ nor the gradient $\nabla \phi \left(\mathbf{p} \right)$ has an analytic expression.

Optimization of $\mathbf{p}$

Approximation by the Numerical Integration

$$\widetilde{\phi}(\mathbf{p}) \triangleq \int_{-\tau_1}^{A+\tau_1} \mathbf{p}^T\mathbf{q}(y)\log_2 \mathbf{p}^T\mathbf{q}(y)\,\mathrm{d}y, \tag{2.20a}$$

$$\nabla\widetilde{\phi}(\mathbf{p}) \triangleq \int_{-\tau_2}^{A+\tau_2} \left(\mathbf{q}(y)\log_2 \mathbf{p}^T\mathbf{q}(y) + \frac{1}{\ln 2}\mathbf{q}(y)\right)\mathrm{d}y. \tag{2.20b}$$

where $\tau_1 > 0$ and $\tau_2 > 0$.

Approximation error can be arbitrarily small with a suffiiently large τ_1 and τ_2.

Inexact Gradient Descent Method

Algorithm 1 Inexact Gradient Descent Method.

Initialization: choose $K \geq 2$, $\lambda_{K-1} \leq 0$, set c_2 ,c_3 as the stopping parameter. set $n = 0$, and choose a random starting point $\mathbf{p}_0 \in \Upsilon$;

```
repeat
    repeat
        Update φ̃(p_n) by (2.17a) and ∇φ̃(p_n) by (2.17b);
        Compute stepsize α_n by Algorithm 1;
        Update p_{n+1} = Proj_Υ( p_n − α_n ∇φ̃(p_n) );
        Let n ← n + 1;
    until ‖p_{n+1} − p_n‖ ≤ c_2;
    λ_K = φ̃(p_n);
until |λ_K − λ_{K−1}| ≤ c_3;
```

- **repeat**
  - **repeat**
    - Update $\widetilde{\phi}(\mathbf{p}_n)$ by (2.17a) and $\nabla\widetilde{\phi}(\mathbf{p}_n)$ by (2.17b);
    - Compute stepsize α_n by Algorithm 1;
    - Update $\mathbf{p}_{n+1} = \mathrm{Proj}_{\Upsilon}\left(\mathbf{p}_n - \alpha_n \nabla\widetilde{\phi}(\mathbf{p}_n)\right)$;
    - Let $n \leftarrow n + 1$;
  - **until** $\left\|\mathbf{p}_{n+1} - \mathbf{p}_n\right\| \leq c_2$;
  - $\lambda_K = \widetilde{\phi}(\mathbf{p}_n)$;
- **until** $|\lambda_K - \lambda_{K-1}| \leq c_3$;

Output: $\mathbf{p}^{\mathrm{opt}} = \mathbf{p}_n$, $K^{\mathrm{opt}} = K$.

Backtracking Line Search

Algorithm 2 Backtracking Line Search

Initialization: choose $\overline{\alpha}, \rho, c \in (0,1)$;

1: **while** $\widetilde{\phi}\left(\overline{\mathbf{p}}_{n+1}\right) \leq \widetilde{\phi}\left(\mathbf{p}_n\right) + c\bar{\alpha}\nabla\widetilde{\phi}(\mathbf{p}_n)^T\left(\overline{\mathbf{p}}_{n+1} - \mathbf{p}_n\right)$, where $\overline{\mathbf{p}}_{n+1} = \mathrm{Pr\,oj}_{\Upsilon}\left[\mathbf{p}_n - \bar{\alpha}\nabla\widetilde{\phi}\left(\mathbf{p}_n\right)\right]$ **do**

2: $\overline{\alpha} \leftarrow \rho\overline{\alpha}$;

3: **end while**

Output: $\alpha_n = \overline{\alpha}$.

Optimality

Theorem 1: (Objective Function)

The inexact objective function $\widetilde{\phi}(\mathbf{p})$ can approach the objective function $\phi(\mathbf{p})$ with an arbitrary small error, i.e.,

$$\left|\phi(\mathbf{p}) - \widetilde{\phi}(\mathbf{p})\right| \leq \varepsilon_1. \quad (2.21)$$

where $\varepsilon_1 > 0$ is a given accuracy. Mathematically, for a given $\varepsilon_1 > 0$, there exists a large parameter $\tau_1 > \sigma$ satisfying

$$\max\left\{\left|2B_1 \mathrm{erfc}\left(\frac{\tau_1}{\sigma\sqrt{2}}\right) + \frac{\tau_1 e^{-\frac{\tau_1^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma\ln 2}\right|, \left|2B_2 \mathrm{erfc}\left(\frac{\tau_1}{\sigma\sqrt{2}}\right) + \frac{(\tau_1 + 2A)\, e^{-\frac{\tau_1^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma\ln 2}\right|\right\} \leq \varepsilon_1, \quad (2.22)$$

where $B_1 \triangleq \frac{1+2\ln\left(\sigma\sqrt{2\pi}\right)}{4\ln 2}$, $B_2 \triangleq \frac{\sigma^2+A^2+2\sigma^2\ln\left(\sigma\sqrt{2\pi}\right)}{\sigma^2 4\ln 2}$.

Optimality

Theorem 2: (Gradient Function)

The inexact gradient function $\nabla\widetilde{\phi}(\mathbf{p})$ can approach the gradient function $\nabla\phi(\mathbf{p})$ with an arbitrarily small gap, i.e., we have

$$\left\|\nabla\phi(\mathbf{p}) - \nabla\widetilde{\phi}(\mathbf{p})\right\| \leq \varepsilon_2. \tag{2.23}$$

where a given accuracy $\varepsilon_2 > 0$. Mathematically, for a given $\varepsilon_2 > 0$, there exists a $\tau_2 \geq \sigma$ satisfying

$$\sqrt{K}\left(\max\left\{\left|2B_1\mathrm{erfc}\left(\frac{\tau_2}{\sigma\sqrt{2}}\right) + \frac{\tau_2 e^{-\frac{-\tau_2^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma\ln 2}\right|,\right.\right.$$
$$\left.\left.\left|2B_2\mathrm{erfc}\left(\frac{\tau_2}{\sigma\sqrt{2}}\right) + \frac{(\tau_1 + 2A)\, e^{-\frac{\tau_2^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma\ln 2}\right|\right\} + \frac{\mathrm{erfc}\left(\frac{\tau_2}{\sigma\sqrt{2}}\right)}{\ln 2}\right) \leq \varepsilon_2. \tag{2.24}$$

Theorem 3: (Convergence Analysis)

For a given K, $\{\mathbf{p}_n\}$ converges to the optimal solution of problem (2.16), and $\{\phi(\mathbf{p}_n)\}$ converges to the corresponding optimal value of problem (2.16).

Simulation Settings

Different Methods

- exhaustive search[10]
- entropy maximization based non-uniform distribution[11]
- Blahut-Arimoto algorithm[12,13]

Simulation Parameter

- $\kappa \triangleq \frac{A}{\mu}$, $\text{SNR} \triangleq \frac{\mu}{\sigma}$
- $\sigma^2 = 1$, $Z \sim \mathcal{N}(0, 1)$

[10] J. G. Smith, "The information capacity of amplitude- and variance- constrained scalar gaussian channels," Inf. Contr., vol. 18, no. 3, pp. 203-219, Feb. 1971.

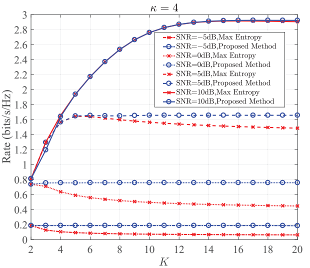
[11] A. A. Farid and S. Hranilovic, "Channel capacity and non-uniform signaling for free-space optical intensity channels," IEEE J. Sel. Areas Commun., vol. 17, no. 9, pp. 1553–1563, Dec. 2009.

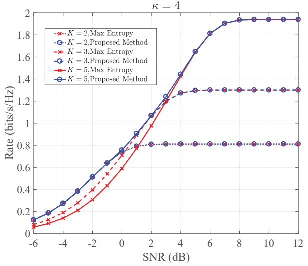
[12] R. Blahut, "Computation of channel capacity and rate-distortion functions," IEEE Trans. Inf. Theory, vol. 18, no. 4, pp. 460–473, Jul. 1972.

[13] J. H. G. Dauwels., On Graphical Models for Communications and Machine Learning: Algorithms, Bounds, and Analog Implementation, ETH PhD thesis, 2006.

Simulation Results

Capacity versus K or SNR

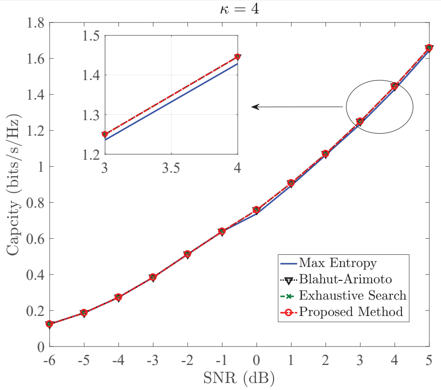




- Inexact Gradient Projection: Higher rate for same K and SNR
- Max-Entropy: rate versus K first increases and then decreases
- As SNR increases, the gap of rate decreases.
- As SNR increases, a large K is needed to achieve the capacity.

Simulation Results

Capacity versus SNR



Simulation Results

Computing Timing

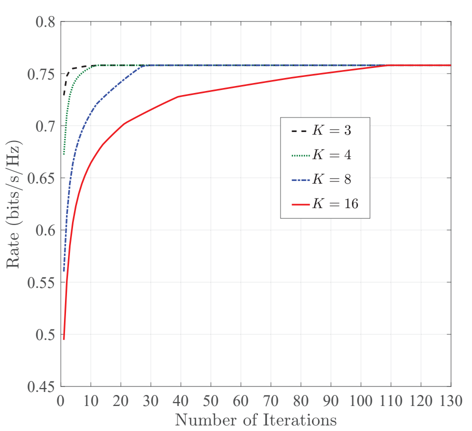
SNR(dB)	CPU time (sec.)		
	Exhaustive Search	Blahut-Arimoto	Proposed Method
-2	2.05	3	1.66
0	809.72	1393.91	51.11
2	16801.8	5820.94	444.08
4		41540.72	542.02

$\kappa = 4$,MATLAB© (R2016b) with 3.4GHz CPUs and 16GB RAM

- Significantly less than the other
- More slowly increases

Convergence

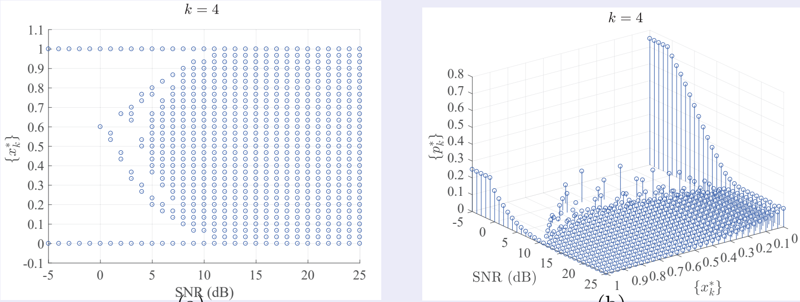
Rate versus the Number of Iterations($\kappa = 4, \mathrm{SNR} = 0\mathrm{dB}$)



- Converge to the rate of the optimal number of points.
- The redundant points can be removed by optimizing $\mathbf{p}$.

Optimal input distribution

Optimal input distribution($\kappa = 4$)



(a) (b)

Figure 2.1: (a) The optimal input positions $\{x_k^*\}$ versus SNR for $\kappa = 4$; (b) The optimal input distribution $\{x_k^*, p_k^*\}$ versus SNR for $\kappa = 4$.

The revealed optimal discrete input distribution is the OOK modulation at low SNR, and the PAM at high SNR.

Conclusion

For achieving channel capacity of VLC,

- based on the inexact gradient projection , we develop a low-complexity method which can converge to the optimal input distribution under both peak and average optical power constraints;
- we prove theoretically the approximations of objection function and its gradient can be arbitrarily accurate by numerical integration;
- we prove theoretically the optimality the inexact gradient projection method and provide numerical results to verify the results;
- we numerically verify hat our method is the most efficient by comparing it with existing methods.

S. Ma, R. Yang, Y. He, S. Lu, F. Zhou, N. Al-Dhahir, and S. Li, "Achieving channel capacity of visible light communication," IEEE Syst. J., pp. 1-12, Mar. 2020.(early access)

Achievable Rate with Closed-form for VLC

- VLC System Model
- ABG Lower Bound
- Simulation Results

VLC System Model

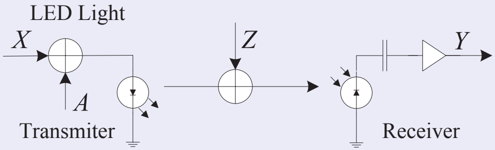


Figure 3.1: The schematic of VLC SISO System.

- IM/DD SISO system
- $X \sim p_X$
- The parameter $A > 0$ is the fixed DC bias to guarantee the transmitted signal $X + A \geq 0$
- $Y = X + A + Z$, where $Z \sim \mathcal{N}\left(0, \sigma^2\right)$

VLC System Model

- Amplitude constraint:

$$-A \leq X \leq A \tag{3.1}$$

- Average optical power:

$$\mathbb{E}\left\{X\right\} = 0,\ P_o = \mathbb{E}\left\{X + A\right\} = A, \tag{3.2}$$

- Average electrical power:

$$\mathbb{E}\left\{X^2\right\} = \varepsilon,\ P_e = \mathbb{E}\left\{(X + A)^2\right\} = \varepsilon + A^2. \tag{3.3}$$

Continuous input distribution

The input X follows continuous distribution and $p_X = f_X(x)$ is the probability density function of X, such that

$$\begin{cases} \int_{-A}^{A} f_X(x)\,dx = 1, \\ f_X(x) \geq 0, x \in [-A, A], \\ f_X(x) = 0, x \notin [-A, A], \end{cases} \tag{3.4}$$

Moreover, the distribution $f_X(x)$ satisfies

$$\mathbb{E}\{X\} = \int_{-A}^{A} x f_X(x)\,dx = 0, \tag{3.5a}$$

$$\mathbb{E}\{X^2\} = \int_{-A}^{A} x^2 f_X(x)\,dx = \varepsilon. \tag{3.5b}$$

Achievable Rate

$$
\begin{aligned}
C^{\mathrm{SISO}} &\geq \max_{f_X(x)} I(X;Y) && (3.6a)\\
&= \max_{f_X(x)} h(Y) - h(Z) && (3.6b)\\
&= \max_{f_X(x)} h(X + A + Z) - h(Z) && (3.6c)\\
&\geq \max_{f_X(x)} \frac{1}{2}\log_2\left(2^{2h(X)} + 2^{2h(Z)}\right) - h(Z) && (3.6d)\\
&= \max_{f_X(x)} \frac{1}{2}\log_2\left(1 + \frac{2^{2h(X)}}{2\pi e\sigma^2}\right), && (3.6e)
\end{aligned}
$$

Achievable Rate

- The optimization problem of maximum differential entropy

$$\min_{f_X(x)} \int_{-A}^{A} f_X(x) \ln f_X(x)\, dx \tag{3.7a}$$

$$\text{s.t.} \int_{-A}^{A} f_X(x)\, dx = 1, \tag{3.7b}$$

$$\int_{-A}^{A} x f_X(x)\, dx = 0, \tag{3.7c}$$

$$\int_{-A}^{A} x^2 f_X(x) dx = \varepsilon. \tag{3.7d}$$

- The differential entropy maximization problem (3.7) is convex.
- We can solve problem (3.7) by lagrangian multiplier method

Achievable Rate

theorem 3.1 (ABG lower bound)

For VLC SISO channel,the ABG lower bound of the channel capacity is given by

$$C^{\mathrm{SISO}} \geq \frac{1}{2}\log_2\left(1+\frac{e^{1+2(\alpha+\gamma\varepsilon)}}{2\pi\sigma^2}\right), \tag{3.8}$$

lower bound can be achieved by X whose distribution is given by

$$f_X\left(x\right) = \begin{cases} e^{-1-\alpha-\beta x-\gamma x^2}, -A \leq x \leq A; \\ 0, \qquad \text{otherwise.} \end{cases} \tag{3.9}$$

ABG Lower Bound

The parameters α, β and γ be the solutions of the following equations

$$T(A) - T(-A) = e^{1+\alpha}, \tag{3.10a}$$

$$\beta\left(e^{A(\beta-\gamma A)} - e^{-A(\beta+\gamma A)} - e^{1+\alpha}\right) = 0, \tag{3.10b}$$

$$e^{A(\beta-\gamma A)}\left((\beta - 2\gamma A)\, e^{-2A\beta} - \beta - 2\gamma A\right) + \left(\beta^2 + 2\gamma\right) e^{1+\alpha} = 4\gamma^2 \varepsilon e^{1+\alpha}, \tag{3.10c}$$

where $T(x) \triangleq \sqrt{\pi}\frac{e^{\frac{\beta^2}{4\gamma}}\operatorname{erf}\left(\frac{\beta+2\gamma x}{2\sqrt{\gamma}}\right)}{2\sqrt{\gamma}}$.

- For convenience, we define parameter $\phi \triangleq \frac{A^2}{\varepsilon}$.
- Note that, when $\phi = 3$, the proposed ABG input distribution is the uniform distribution.

Simulation Results

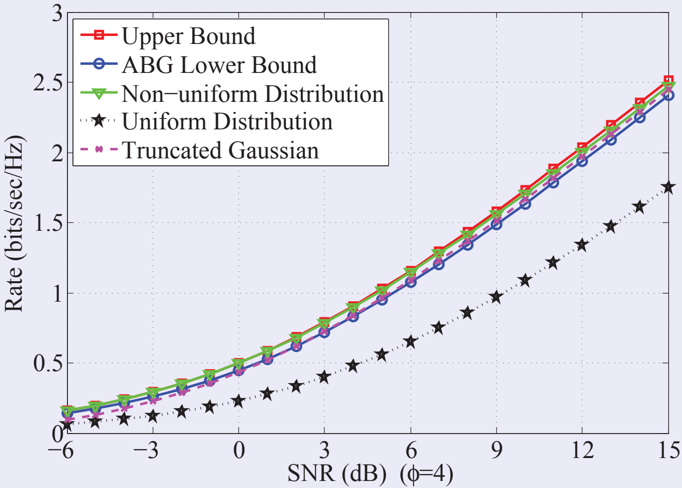


Figure 3.2: The achievable rate (bits/sec/Hz) versus SNR (dB) with $\phi = 4$.

Optimal Power Allocation for Mobile NOMA VLC

- System Model of NOMA VLC
- Achievable Rates for Mobile Users of NOMA
- Optimal Power Allocation Scheme for Mobile Users of NOMA

System Model

Downlink of NOMA VLC

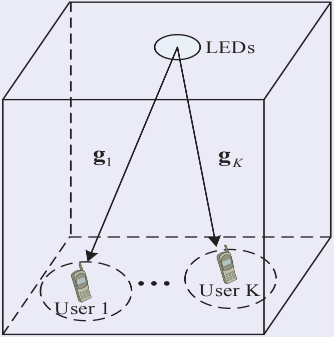


Figure 4.1: The schematic of downlink of NOMA VLC.

- Assume that the locations area of users can be bounded by circles;

The geometry of mobile users

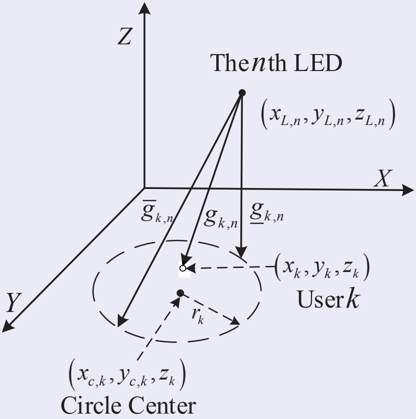


Figure 4.2: The geometry of mobile users

Channel Model of Mobile Users

- The user k is located within the circular area with center $(x_{c,k}, y_{c,k}, z_k)$ and radius r_k, i.e.,

$$\mathcal{R}_k \triangleq \left\{ (x_k, y_k) \middle| 0 \le (x_k - x_{c,k})^2 + (y_k - y_{c,k})^2 \le r_k^2 \right\}. \tag{4.1}$$

- Then, the channel gain between the nth LED and user k is given as

$$g_{k,n} = \frac{(m+1)\, A_R}{2\pi d_{k,n}^2} \left(\frac{z_{L,n} - z_k}{d_{k,n}} \right)^{m+1} = \frac{\beta (z_{L,n} - z_k)^{m+1}}{d_{k,n}^{m+3}}, \tag{4.2}$$

Channel Model of Mobile Users

- Furthermore, the CSI uncertainty vector $\Delta\mathbf{g}_k$ can be characterized by an ellipsoidal region:

$$\mathcal{G}_{\mathrm{k}} \triangleq \left\{\Delta\mathbf{g}_k | \Delta\mathbf{g}_k^T \mathbf{C}_k \Delta\mathbf{g}_k \leq \upsilon_k\right\}, \tag{4.3}$$

where $\mathbf{C}_k = \mathbf{C}_k^T \succeq \mathbf{0}$ controls the extension of the ellipsoid

- Before user k decoding the signal s_i, the residual received signal is given as

$$y_{k,i}^{\mathrm{Mob,SIC}} = \mathbf{g}_k^T \mathbf{w}_i s_i + \sum_{l=1}^{i-1} \Delta\mathbf{g}_k^T \mathbf{w}_l s_l + \sum_{m=i+1}^{K} \mathbf{g}_k^T \mathbf{w}_m s_m + \mathbf{g}_k^T \mathbf{b} + n_k, \tag{4.4}$$

Achievable Rate of Mobile Users

- The lower bound of $R_{k,i}^{\mathrm{Mob}}$ is given by

$$R_{k,i}^{\mathrm{Mob}} = \max_{\{f_i(x_i)\}} I\left(x_i; y_{k,i}^{\mathrm{Mob,SIC}}\right) \tag{4.5a}$$

$$= \max_{\{f_i(x_i)\}} h\left(y_{k,i}^{\mathrm{Mob,SIC}}\right) - h\left(y_{k,i}^{\mathrm{Mob,SIC}} | x_i\right) \tag{4.5b}$$

$$\geq \frac{1}{2}\log_2 \frac{2\pi\sigma_k^2 + \sum_{j=i}^{K} \left|\mathbf{g}_k^T \mathbf{w}_j\right|^2 \tau_j + \sum_{l=1}^{i-1} \left|\Delta\mathbf{g}_k^T \mathbf{w}_l\right|^2 \tau_l}{2\pi\left(\sigma_k^2 + \sum_{m=i+1}^{K} \left|\mathbf{g}_k^T \mathbf{w}_m\right|^2 \varepsilon_m + \sum_{n=1}^{i} \left|\Delta\mathbf{g}_k^T \mathbf{w}_n\right|^2 \varepsilon_n\right)}, \tag{4.5c}$$

where $\tau_j \triangleq e^{1+2(\alpha_j+\gamma_j\varepsilon_j)}$

Optimal Power Allocation for Mobile Users

Problem Formulation

- The optimal power allocation problem of mobile users can be formulated as

$$\min_{\{\mathbf{w}_k\}_{k=1}^{K}} \sum_{k=1}^{K} \varepsilon_k \|\mathbf{w}_k\|^2 \tag{4.35a}$$

$$\text{s.t. } R_{k,i}^{\mathrm{L,Mob}} \geq \bar{R}_i, 1 \leq i \leq k \leq K, \tag{4.35b}$$

$$\sum_{k=1}^{K} A_k \left|w_{k,n}\right| \leq b, \forall n \in \mathcal{N}, \tag{4.35c}$$

$$\mathbf{g}_k = \widehat{\mathbf{g}}_k + \Delta\mathbf{g}_k, \Delta\mathbf{g}_k \in G_{\mathrm{k}}, \forall k \in \mathcal{K}. \tag{4.35d}$$

Optimal Power Allocation for Mobile Users

Optimal Power Allocation

- Let define $\mathbf{W}_k = \mathbf{w}_k \mathbf{w}_k^T$, and then constraint (4.35b) can be reformulated as

$$\Delta \mathbf{g}_k^T \widehat{\mathbf{W}}_{k,i} \Delta \mathbf{g}_k + 2\Delta \mathbf{g}_k^T \widetilde{\mathbf{W}}_{k,i} \widehat{\mathbf{g}}_k + \widehat{\mathbf{g}}_k^T \widetilde{\mathbf{W}}_{k,i} \widehat{\mathbf{g}}_k \geq c_{k,i}, \tag{4.36}$$

where

$$\widehat{\mathbf{W}}_{k,i} \triangleq \sum_{j=1}^{K} \tau_j \mathbf{W}_j - 2^{2\bar{R}_i+1}\pi \sum_{n=1}^{i} \varepsilon_n \mathbf{W}_n - 2^{2\bar{R}_i+1}\pi \Gamma_{k,i} \sum_{m=i+1}^{K} \varepsilon_m \mathbf{W}_m, \tag{4.37}$$

$$\widetilde{\mathbf{W}}_{k,i} \triangleq \sum_{j=i}^{K} \tau_j \mathbf{W}_j - 2^{2\bar{R}_i+1}\pi \Gamma_{k,i} \sum_{m=i+1}^{K} \varepsilon_m \mathbf{W}_m, \tag{4.38}$$

$$c_{k,i} \triangleq \left(2^{2\bar{R}_i} - 1\right) 2\pi \sigma_k^2. \tag{4.39}$$

Optimal Power Allocation for Mobile Users

SDR

- Furthermore, by ignoring the rank-one constraint for all $\mathbf{W}_k$'s, the SDR of problem (4.40) is given by

$$\min_{\{\mathbf{W}_k\}_{k=1}^{K}} \varepsilon_k \mathrm{Tr}\left(\mathbf{W}_k\right) \tag{4.40a}$$

$$\text{s.t. } \Delta\mathbf{g}_k^T \widehat{\mathbf{W}}_{k,i} \Delta\mathbf{g}_k + 2\Delta\mathbf{g}_k^T \widetilde{\mathbf{W}}_{k,i} \widehat{\mathbf{g}}_k + \widehat{\mathbf{g}}_k^T \widetilde{\mathbf{W}}_{k,i} \widehat{\mathbf{g}}_k \geq c_{k,i}, \quad 1 \leq i \leq k \leq K, \tag{4.40b}$$

$$\sum_{k=1}^{K} \mathrm{Tr}\left(\mathbf{W}_k \mathbf{e}_n \mathbf{e}_n^T\right) \leq \frac{b^2}{A_k^2}, \forall n \in \mathcal{N}, \tag{4.40c}$$

$$\mathbf{W}_k \succeq \mathbf{0}, \ \forall k \in \mathcal{K}, \tag{4.40d}$$

$$\Delta\mathbf{g}_k \in \mathcal{G}_{\mathrm{k}}, \ \forall k \in \mathcal{K}. \tag{4.40e}$$

Optimal Power Allocation for Mobile Users

S-Lemma

- By using $\mathcal{S}$-lemma,we obtain the following conservative approximation of problem (4.40) as follows

$$
\begin{aligned}
&\max_{\{\mathbf{W}_k\}} \sum_{k=1}^{K} \varepsilon_k \mathrm{Tr}\left(\mathbf{W}_k\right) && (4.41a)\\
&\text{s.t.} \begin{bmatrix} \widehat{\mathbf{W}}_{k,i} + \lambda_{k,i}\mathbf{C}_k & \widetilde{\mathbf{W}}_{k,i}\widehat{\mathbf{g}}_k, \\ \widehat{\mathbf{g}}_k^T \widetilde{\mathbf{W}}_{k,i} & \widehat{\mathbf{g}}_k^T \widetilde{\mathbf{W}}_{k,i}\widehat{\mathbf{g}}_k - c_{k,i} - \lambda_{k,i}\upsilon_k \end{bmatrix} \succeq \mathbf{0},\\
&\quad 1 \le i \le k \le K, && (4.41b)\\
&\quad \sum_{k=1}^{K} \mathrm{Tr}\left(\mathbf{W}_k \mathbf{e}_n \mathbf{e}_n^T\right) \le \frac{b^2}{A_k^2}, \forall n \in \mathcal{N}, && (4.41c)\\
&\quad \mathbf{W}_k \succeq \mathbf{0},\ \forall k \in \mathcal{K}, && (4.41d)
\end{aligned}
$$

which is a convex SDP.

Simulation Results

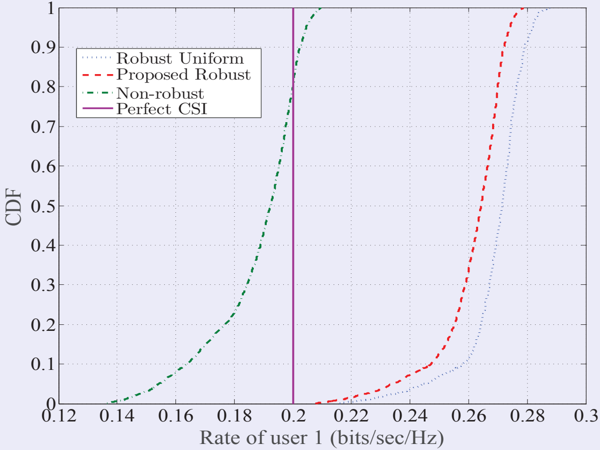


Figure 4.3: CDF of rate over 20000 random channel realizations of user 1.

Simulation Results

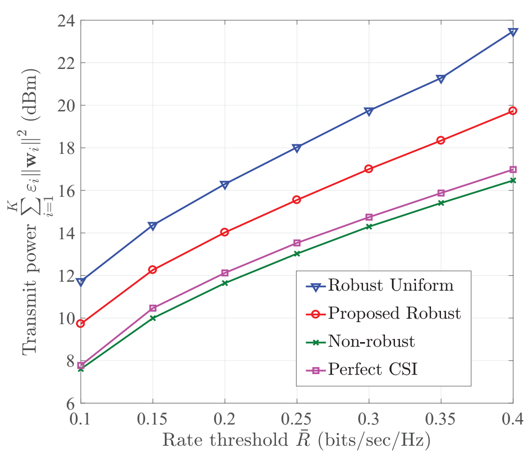


Figure 4.4: Transmit power $\sum_{i=1}^{K} \varepsilon_i \|\mathbf{w}_i\|^2$ (dBm) versus rate threshold $\bar{R}$.

Simulation Results

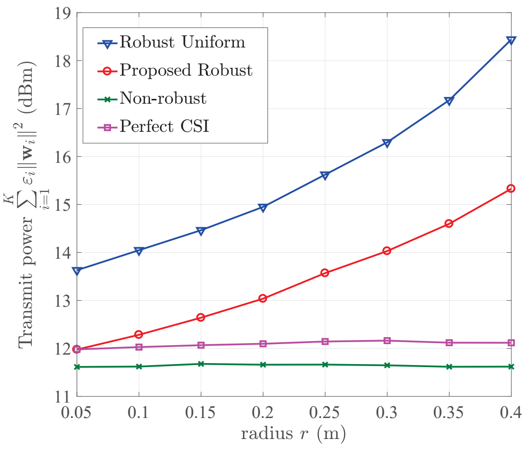


Figure 4.5: Transmit power $\sum_{i=1}^{K} \varepsilon_i \|\mathbf{w}_i\|^2$ (dBm) versus radius r (m).

Summary

Summary

- We proposed an efficient inexact gradient descent method to obtain the channel capacity-achieving input distribution of VLC.
- We developed a closed-form ABG lower bound for VLC channels;
- We proposed a robust power allocation scheme for mobile users scenario in NOMA VLC networks.

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- Shuai Ma, Hang Li, Yang He, Ruixin Yang, Songtao Lu, Wen Cao, and Shiyin Li, "Capacity Bounds and Interference Management for Interference Channel in Visible Light Communication Networks," *IEEE Transcations on Wireless Communications*, vol. 18, no. 1, pp. 182-193, Nov 2019.
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Thanks For Your Attention!